

# Miniaturization and Optimization of Planar Microwave Filters Based on Metamaterials

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**Abstract**— Several possibilities for the implementation of transmission line filters based on metamaterials are presented and discussed in this paper. The proposed designs take advantage of different aspects of metamaterial behaviour to achieve competitive filter performance. A combination of right and left-handed transmission lines is used for a design of miniature stepped-impedance resonators and filters either for a suppression of spurious responses or for achievement of a dual-band response. On the other hand, an analytical method for the synthesis of wideband band pass filters is exposed. In both cases, the dual transmission line approach has been considered.

## I. INTRODUCTION

Since the first experimental demonstration of a double negative medium (originally studied by Veselago [1]) by Smith *et al.* in 2000 [2, 3], there has been an intense activity from the scientific community devoted to the analysis and development of these artificial media or metamaterials. As a consequence of such intense effort, the engineering applications of metamaterials, especially in the field of planar technologies, is nowadays a reality (see these books on the field for a complete review of applications [4, 5, 6]). The possibility to implement artificial transmission lines (TL) consisting on left handed (LH), composite right/left handed (CRLH) or cascaded right handed/left handed RH/LH structures offers new and very interesting perspectives for the implementation of many devices such as dual-band components, enhanced-bandwidth devices or leaky wave antennas, among others.

It is the aim of this work to use these artificial transmission lines implemented by means of L-C loaded lines for the design of compact filters in planar technology. In section II, some examples of the application of the RH/LH TL to the design of stepped impedance resonator (SIR) are exposed. These results point out that difference in dispersion characteristics of the LH and RH TL gives additional degrees of freedom and makes it possible to design microwave devices with unique characteristics based on a combination of the LH and RH TL sections. In the section III, a broadband and compact LH microstrip filter consisting on capacitively coupled grounded strips is proposed, and analytical expressions for the design are provided.

## II. DESIGN OF STEPPED IMPEDANCE RESONATORS BASED ON A COMBINATION OF RIGHT/LEFT-HANDED TRANSMISSION LINE SECTIONS

A resonator on a dispersionless transmission line section has a set of resonant frequencies, which are multiples. In many practical applications it is necessary to reject the second and third harmonics. The popular way to suppress the higher harmonics is using a SIR [7, 8]. In order to obtain more freedom in a design of harmonic free planar resonator, we suggest to use a SIR based on a cascaded connection of the LH TL and RH TL sections. In contrast to the ordinary SIR on RH transmission line sections, the novel one gives a possibility to change the resonant frequency of higher modes by using different dispersion characteristics of the RH and LH transmission lines. Another problem is a design of a dual-band resonator for band pass filter applications [9, 10]. In this case using the RH/LH SIR is also beneficial. In practice, an artificial lumped-element implementation of the LH TL has to be used while the RH TL can be realized as either a lumped-element structure or a real transmission line section. That makes possible to decrease the dimensions of the resonators and filters. We consider the both problems: a suppression of higher harmonics in the resonator and a design of dual-band resonator on a cascaded connection of the LH TL and RH TL sections.

### A. The stepped-impedance resonator and filter based on a cascaded connection of the LH TL and RH TL sections

The open-ended SIR based on a series connection of RH and LH transmission line sections with characteristic impedance  $Z_R$  and  $Z_L$  is shown in Fig. 1. The electrical length of RH and LH TL sections are  $\theta_R > 0$  and  $\theta_L < 0$  correspondingly. The equations for the fundamental mode and the first harmonic response are as follows:

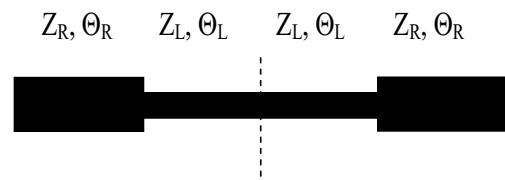


Fig. 1 Equivalent circuit of the LH/RH TL resonator

$$\tan(\theta_{0L}) = -\frac{Z_R}{Z_L} \cot(\theta_{0R}), \quad (1)$$

$$\tan\left(\theta_{0L} \frac{\omega_0}{\omega_1}\right) = \frac{Z_L}{Z_R} \tan\left(\theta_{0R} \frac{\omega_1}{\omega_0}\right). \quad (2)$$

Here  $\theta_{0R}$  and  $\theta_{0L}$  are the electrical lengths at the fundamental resonant frequency  $\omega_0$ ,  $\omega_1$  is the resonant frequency of the first harmonic response.

The RH TL is performed as a natural TL, whereas the LH TL section is made as a symmetric T-section of a high-pass circuit. In line with (1) and (2), the shorter the length of the RH TL section, the higher is the resonant frequency  $\omega_1$ . The resonance characteristics are also influenced by the chosen value of the characteristic impedance of the RH TL section: the higher is the ratio  $Z_R/Z_L$ , the higher is  $\omega_1$  and the more effective is the suppression of the higher harmonics. The RH/LH SIR was designed as fully integrated microwave circuit and fabricated using multilayer ceramic technology [11]. The fundamental mode frequency for this resonator was 3 GHz and the first spurious response was observed at 11.6 GHz. The area occupied by the RH/LH TL resonator was  $6.15 \times 3.65 \text{ mm}^2$ .

The LH/RH TL SIR was used for a design of a miniature band-pass filter with suppressed spurious response [12]. The 2-pole filter was designed for the fundamental mode frequency  $f_0 = 1.56 \text{ GHz}$ . Simulated and measured characteristics are presented in Fig. 2. The first spurious response is observed at about 5 GHz. The filter area is  $15 \times 15 \text{ mm}^2$ .

#### B. The dual-band filter based on a cascaded connection of the LH TL and RH TL sections

The same equivalent diagram of the SIR was analysed as the dual-band resonator with suppressed higher harmonics. In this case, a rather long RH TL section should be used. In order to miniaturize the device, the RH TL was designed as a cascaded connection of a symmetric  $\Pi$ -section of a low-pass circuit and a natural TL section. The equivalent diagram of the dual-band SIR is depicted in Fig. 3. The dual-band SIR with resonances at  $f_0 = 2 \text{ GHz}$  and  $f_1 = 3 \text{ GHz}$  was realized using multilayer LTCC technology. According to the full-wave simulation the first spurious response was not observed up to 9 GHz.

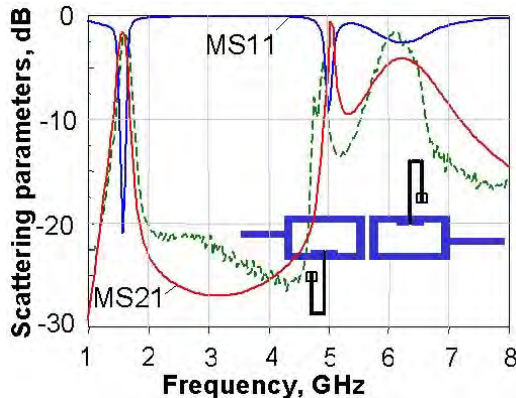


Fig. 2 Simulated (solid lines) and measured (dashed line) characteristics of the two-pole filter based on RH/LH TL SIR ( $\theta_{0R} = \pi/6$ ,  $Z_R/Z_L = 1.1$ )

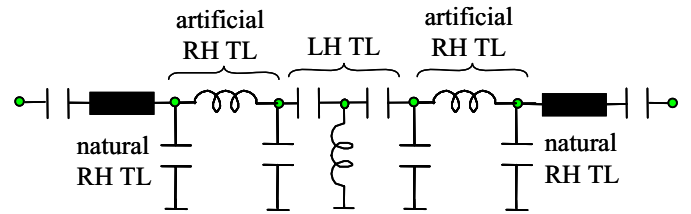


Fig. 3 Equivalent diagram of the dual-band RH/LH TL SIR.

The 2-pole filter was designed as a cascaded connection of the LH/RH TL dual-band SIRs. The miniature multilayer LTCC implementation of the filter is shown in the Fig. 4. a. The filter area is  $20 \times 8 \text{ mm}^2$ . The filter performance obtained by the equivalent circuit modelling (dashed line) and results of the full-wave simulation (solid line) are shown in a comparison in Fig. 4. b. The filter bandwidths are 150 MHz. The reflection coefficients in the pass bands are better than 16 dB. Isolation between the operational bands is not worse than 20 dB. In the wide frequency range (see inset) the spurious response is not observed up to 7 GHz.

### III. LEFT HANDED TRANSMISSION LINES BASED ON COUPLED METALLIC GROUNDED STRIPS FOR BROADBAND BAND PASS FILTER DESIGN

In this section, a transmission line with LH characteristics is proposed for the design of band pass filters with broad bandwidth and compact dimensions. The structure can be described by a lumped element equivalent circuit similar to that of the LH TL section of Fig. 3 (which is the dual transmission line introduced and developed almost simultaneously by Iyer [13], Oliner [14] and Caloz [15]). This dual line exhibits high pass behaviour, so that an additional element is required to achieve band pass characteristics. This additional element is simply a capacitance parallel connected to the shunt inductance.

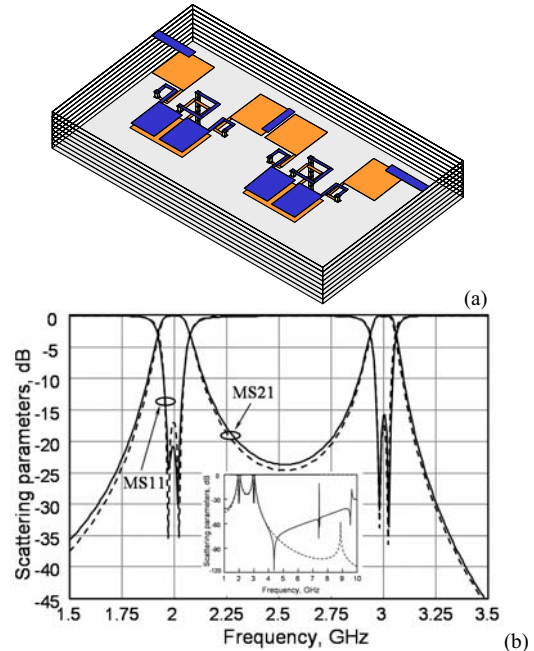


Fig. 4 Multilayer structure (a) and simulated performance (b) of the 2-pole filter based on the dual-band RH/LH TL SIR. Inset 1-10 GHz.

### A. Physical Structure

The LH TL structure, inspired in the mushroom structure of Sievenpiper [16, 17] is depicted in figure 5 and it consists on a cascade of wide transmission line sections coupled through gap capacitors and grounded by means of a pair of vias symmetrically placed. The equivalent circuit model for the basic cell is depicted in Fig 5.b, where the vias are modelled by the inductance  $L_v$ ,  $C_g$  accounts for the series gaps, and the capacitance  $C$  is the line capacitance. The line inductance can be neglected in this model since the line sections are very wide. Thus the forward wave transmission band is expected to appear at frequencies much beyond the region of interest. This is of interest in filters that only exploit the left handed band, though other filter design strategies have been proposed where both the left handed and the right handed regions are used under balance conditions [18, 19]. As can be seen to the light of the circuit model, it consists on a dual transmission line model with the presence of an additional shunt connected capacitance,  $C$ , as desired.

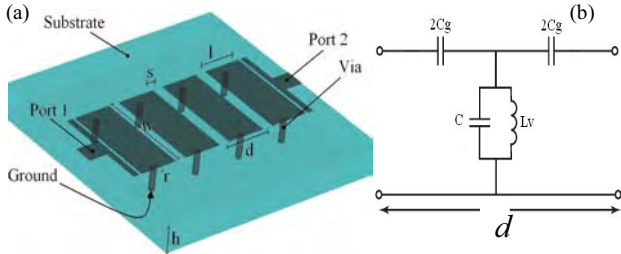


Figure 5: Proposed left handed transmission line structure (a) and equivalent circuit model of the unit cell (b).

### B. Design

In the structure of Fig. 5.a, the parameters of the unit cell can be obtained from analytical expressions derived from their physical interpretation. The capacitance  $C$  is formed by the contributions of two different shunt capacitances: The parallel-plate capacitance of the microstrip sections and the shunt capacitance owing to microstrip discontinuities as can be observed in equation (3).

$$C = C_{\text{parallel-plate}} + 2C_p \quad (3)$$

$C_{\text{parallel-plate}}$  can be obtained from the well known expression (4)

$$C_{\text{parallel-plate}} = \frac{\epsilon_0 \epsilon_r W l}{h} \quad (4)$$

where  $\epsilon_r$  is the dielectric constant,  $\epsilon_0$  is the dielectric permittivity of vacuum,  $h$  is the substrate thickness,  $W$  is the width of the strip and  $l$  is its length. On the other hand,  $C_p$  can be estimated using the eq. (5) [20,21]:

$$C_p = \frac{12 \cdot W}{2} \left( \frac{\epsilon_r}{9.6} \right)^{0.9} \left( \frac{s}{W} \right)^{m_e} e^{k_e} \quad (5)$$

where  $s$  is the gap separation and  $m_e$  and  $k_e$  are parameters which values depend of the ratio  $W/h$  and should be estimated from the available numerical results [22].

The other capacitance of the equivalent circuit model shown in the figure 5.b ( $C_g$ ) can be interpreted as the capacitance produced by a microstrip discontinuity and can be estimated using the expression (6) [20,21,22].

$$C_g = \frac{W}{2} \left( \frac{\epsilon_r}{9.6} \right)^{0.8} \left( \frac{s}{W} \right)^{m_0} e^{k_0} - \frac{1}{4} 12W \left( \frac{\epsilon_r}{9.6} \right)^{0.9} \left( \frac{s}{W} \right)^{m_e} e^{k_e} \quad (6)$$

where again,  $k_0$ ,  $m_0$ ,  $k_e$ , and  $m_e$  are parameters which values depends of the ratio  $W/h$  and can be estimated from the available numerical results for each particular case. Finally  $L_v$  is the inductance introduced by a cylindrical via hole which is function of the diameter and the substrate thickness. It is ruled by the expression (7) [23]

$$L_v = \frac{\mu_0}{2 \cdot \pi} \left[ h \cdot \ln \left( \frac{h + \sqrt{r^2 + h^2}}{r} \right) + \frac{3}{2} \left( r - \sqrt{r^2 + h^2} \right) \right] \quad (7)$$

where  $r$  is the vias radius.

As far as the proposed device can be considered a periodic structure, standard analysis technique based on the  $ABCD$  transmission matrix [24] can be applied to obtain the dispersion relation ruled by the expression (8).

$$\cos \beta d = 1 + \frac{\omega^2 C \cdot L_v - 1}{2\omega^2 C \cdot L_v} \quad (8)$$

The allowed LH pass band of this structure is delimited by the following frequencies shown in the equation (9) and equation (10).

$$f_H = \frac{1}{2\pi} \sqrt{\frac{1}{C \cdot L_v}} \quad (9)$$

$$f_L = \frac{1}{2\pi} \sqrt{\frac{1}{(4C_g + C) \cdot L_v}} \quad (10)$$

### C. Fabrication and measurement results

In order to show the viability of the proposed device, an illustrative prototype has been fabricated and tested. The values of the parameters showed in the figure 5.a are:  $l=2$  mm,  $W=5$  mm,  $s=0.6$  mm,  $r=2$  mm and  $d=2.6$  mm. As a consequence of this choice, the equivalent circuit parameters are:  $C_g=0.41$  pF;  $C=1.84$  pF and  $L_v=0.17$  nH.

The left handed behaviour is demonstrated by comparing the phase of  $S_{21}$  ( $\phi_{S21}$ ) for different number of cells. By increasing the number of cells,  $\phi_{S21}$  increases, which is indicative of negative values of the phase velocity and  $\beta$  in the pass band region.

The measured insertion and return losses of the structure for different number of cells are also depicted in figure 6. In spite of the large number of stages (which makes frequency selectivity to be very high), losses are relatively small.

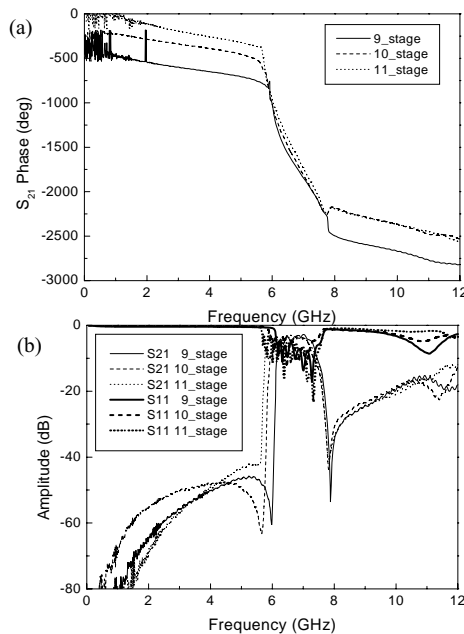


Figure 6 (a) measured  $S_{21}$  phase parameter of structures with different number of stages. (b) Measured S-parameters.

The device is compact (area:  $104 \text{ mm}^2$  for a 10 stage device) and the design procedure is very simple, since equations that link physical to electrical parameters have been proposed.

#### IV. CONCLUSIONS.

The results exposed in the paper point out specific applications in filters in which the use of metamaterial concepts allows to improve critical aspects of the device performance.

In the case of the results exposed on the section II, it is shown that the difference in dispersion characteristics of the LH and RH TL gives additional degrees of freedom and makes it possible to design microwave devices with unique characteristics based on a combination of the LH and RH TL sections.

In the case of the section III, it is demonstrated that a LH line implemented in microstrip technology by loading the line with series gaps and vias holes is useful for the design of band pass filters with wide bandwidth.

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